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The physical nature of weak shock wave reflection

By BERIC W. SKEWS AND JASON T. ASHWORTH

Department of Mechanical, Industrial, and Aeronautical Engineering, University of the Witwatersrand, Johannesburg, South Africa

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For weak shock waves and small wedge angles the application of three-shock (von Neumann) theory gives no physically realistic solutions and yet experiments clearly show a pattern of reflection of three shocks meeting at a triple point. This disagreement is referred to as the von Neumann paradox, and the reflection pattern as von Neumann reflection (vNR). Some recent numerical computations have indicated the existence of an expansion wave immediately behind the reflected wave as originally suggested by Guderley over fifty years ago. Furthermore, a recent solution of the inviscid transonic equations has indicated the possible existence of a very small, multi-wave structure immediately behind the three-shock confluence. A special shock tube has been constructed which allows Mach stem lengths to be obtained which are more than an order of magnitude larger than those obtainable in conventional shock tubes. Schlieren photographs do indeed show a structure consisting of an expansion wave followed by a small shock situated behind the confluence point, with some indication of smaller scale structures in some tests. This indicates that some of the earlier models of vNR, in the parameter space tested, are incorrect. The size of the region influenced by this small wave system is about 2 % of the Mach stem length and it is therefore not surprising that it has not been detected before in conventional shock tube facilities.

1. Introduction

When a shock wave reflects off a wedge a number of different reflection patterns are generated. In regular reflection (RR) the incident shock wave (I) and the reflected shock wave (R) meet at the surface, as shown in figure 1, and is typical for large wall angles. The type of reflection depends on (M_s, γ, θ_w) parameter space, where M_s, γ and θ_w are the incident shock wave Mach number, gas specific heat ratio and flow deflection angle respectively. Reducing the wall angle causes regular reflection to transition, for moderate Mach numbers, into simple Mach reflection (MR). The reflection point detaches from the surface and an additional shock wave, the Mach stem (M), connects the incident and reflected shock waves to the surface. The point of intersection of the three shock waves is known as the triple point (T), which for a plane wedge traces a trajectory at an angle χ to the wedge surface. The slipstream (S) is a thermodynamic contact discontinuity which separates the gas that has passed through the incident and reflected shock waves from the gas that has passed through the Mach stem. For weak shocks and small wall angles there is no apparent discontinuity in slope between the incident shock and the Mach stem, and the slipstream, if visible at all, becomes ill-defined. This pattern is referred to as von Neumann reflection (vNR).

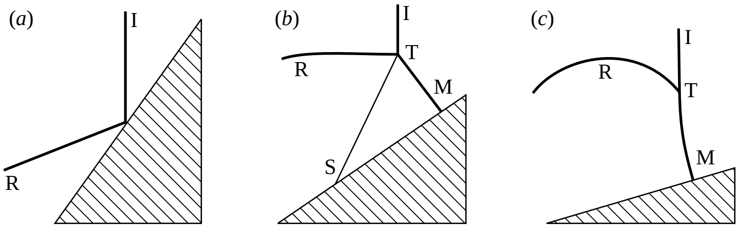

FIGURE 1. Shock wave reflection geometries. (*a*) Regular reflection, (*b*) Mach reflection, (*c*) von Neumann reflection.

The basic equations that describe regular and Mach reflection, known as two- and three-shock theory respectively, were formulated by von Neumann (1943). The analysis applies the oblique shock wave equations for the regions in close proximity to the reflection or triple point, with relevant geometrical and boundary conditions. The basic assumptions of the theory are that the fluid is inviscid, that all the waves are infinitely thin and plane, and that they separate regions having uniform properties. Good agreement between theory and experiment is achieved for regular and Mach reflection except that regular reflection persists slightly beyond the theoretical limit, which has been shown to be due to thermal and viscous boundary layers on the wedge surface. However for von Neumann reflection the theory is inadequate and this discrepancy has been the subject of considerable investigation over many years. It is referred to as the von Neumann paradox, a label first used by Birkhoff (1950). These investigations have taken two different approaches: analytical and computational, some concentrating on the overall reflection pattern and others on the flow in the immediate vicinity of the triple point.

Some of the analytical models concentrate on the profile of the reflected wave with the measure of their success being the agreement with the triple-point trajectory angle. They thus do not say anything about the flow at the three-shock confluence. These models range from extensions of the linearized solution of Lighthill (1949) such as that of Sakurai & Takayama (2004) who included nonlinearities through a singular perturbation technique, to the work of Sandeman (2000) who treated the reflected wave as a blast wave reflecting off the face of the wedge. There have been a number of numerical studies that have mimicked the experimental data such as the triple-point trajectory angle and reflected-shock shape and position, but which have not thrown any further light on the reasons for the paradox. An influential study of this type is that of Colella & Henderson (1990). This was a high-resolution numerical study using the Euler equations combined with experimental results. Good agreement was obtained with the triple-point trajectory angles but differences were found in the wave reflection angles. They identified the von Neumann reflection as being different from Mach reflection. Their conclusion was that transition from simple Mach reflection to von Neumann reflection occurred when the three-shock theory predicted an angle between the reflected wave and the slipstream of $\pi/2$, as beyond this value the theory and experiment diverged. Furthermore they concluded that the incident wave and Mach stem constituted a smooth curve without a discontinuity in slope and that the reflected wave is not a shock but a compression wave in the vicinity of the shock confluence. This transition condition and nature of the reflected wave soon became embedded in the literature as describing the properties of von Neumann

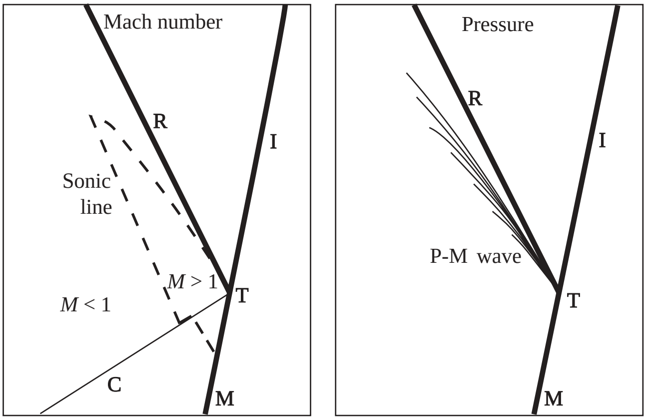

Figure 2. High-resolution study demonstrating the expansion wave (Vasil'ev & Kraiko 1999).

reflection. The detailed nature of the flow field was, however, not examined, nor has any experimental evidence emerged to support their hypothesis.

A more detailed study of the flow in the vicinity of the shock confluence was clearly necessary to resolve the paradox. It is interesting to note that in 1947 Guderley (see also Guderley 1962) proposed the existence of an expansion fan centred at the triple point, and positioned immediately behind the reflected wave. Vasil'ev & Kraiko (1999) give an interesting suggestion of why this work was not followed up for over half a century. The main reason is that the region where this structure occurs is very small, "..beyond the scope of currently available experimental facilities..[and]... much less possible to resolve in shock-capturing computations without using special treatment".

Some workers, e.g. Olim & Dewey (1992) tried to find solutions by relaxing the condition that the flow coming from either side of the triple point should not be parallel, and even that the pressures could be different. They also assumed that the Mach stem was the arc of a circle centred on the reflecting surface. Better agreement was obtained than from the three-shock theory but direct evidence to support these assumptions is scant. Sandeman (1997) took the extensive experimental results of these workers, and those of Sasoh, Takayama & Saito (1995), together with his own predictions (Sandeman 2000) to calculate wave angles at the triple point using both the three-shock and four-wave geometry. It was concluded that both the three-shock and the Guderley model are not consistent with both experiment and simulations but he also suggested that this could be due to lack of sufficient resolution.

Vasil'ev & Kraiko (1999) indicated, through a high-resolution numerical study using the Euler equations, that the four-wave pattern suggested by Guderley does, in fact, exist. They show that for a wall angle of 12.5° and a Mach number of 1.47 the angle between the slipstream and the reflected shock exceeds $\pi/2$ and there is a fan of rarefaction waves centred at the triple point. The subsonic flow immediately behind the reflected wave is convergent, passes through sonic velocity, and a small supersonic patch results, as originally proposed by Guderley (1947). Their result is shown in figure 2 with the sonic line dotted. With the wedge angle increased to 20° the supersonic region shrank to such a degree that it could no longer be resolved even with additional grid refinement. The angle between the reflected shock and the slipstream again exceeded the $\pi/2$ value. In one example they demonstrated that the extent of the region showing the fourth wave is several thousandths of the

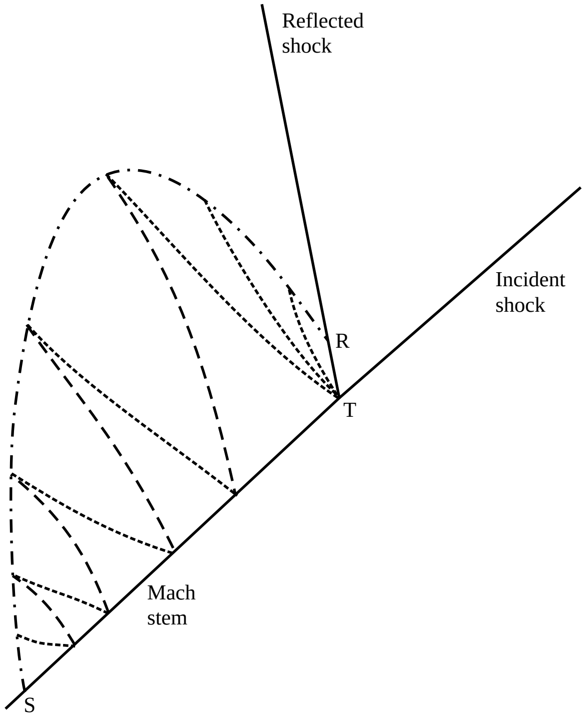

FIGURE 3. The Guderley (1947) structure confirmed in Hunter & Brio (2000).

dimension of the perturbed flow, thereby reinforcing the assertion that the reason that Guderley's work had not received due recognition was simply because of insufficient resolution being achievable. This work also suggests a number of distinctions of flow types within the domain of the von Neumann paradox, and finds no evidence of the reflected wave being a compression wave as suggested in Colella & Henderson (1990).

On the theoretical front Brio & Hunter (1992) started examining these flows by examining Mach reflection using the unsteady transonic small-disturbance equations. Čanić & Keyfitz (1996) proposed a new theoretical approach which suggested the existence of a complex flow structure at the triple point. Further numerical solutions by Hunter & Brio were presented in 1997 indicating the existence of a supersonic patch. A similar numerical study to that of Vasil'ev & Kraiko (1999) was undertaken by Zakharian *et al.* (2000) with very similar findings. This agreed with further solutions to the unsteady transonic small-disturbance equations obtained by Hunter & Brio (2000). The solution that emerged is very similar to that put forward by Guderley (1947) and is shown graphically in figure 3. The dotted lines are minus characteristics, the dashed lines plus characteristics, and the chain dotted line the sonic line. R and S are the sonic points on the reflected and Mach shocks, although following Guderley it is likely that R and T coincide and the triple point is sonic. Furthermore, in analogy to the supersonic patch on a transonic airfoil being terminated by a shock, the authors speculated on the possible existence of a small shock existing behind the supersonic patch in this weak shock reflection case as well, and also the possibility of a series of such patches. This was subsequently shown to be so by Tesdall & Hunter (2002) through the development of a new numerical scheme. Their result (figure 4) shows a series of triple points and supersonic patches, terminated by shocks, existing between

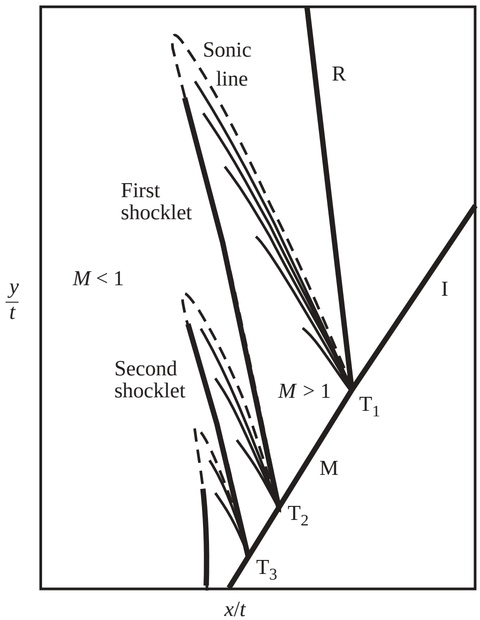

FIGURE 4. The complex flow of Tesdall & Hunter (2002).

the sonic line and the Mach shock. This novel result was the trigger for the current attempt to obtain experimental data.

On the issue of scaling, the transonic small-disturbance reflection problem treated by Tesdall & Hunter (2002) depends on a single order-one transonic similarity parameter $a = \theta/\sqrt{2(M^2-1)}$ where the wedge angle, θ, is small and of the order of $\sqrt{M^2-1}$ rad and M is the incident Mach number, close to 1. If the characteristic length of the problem is taken as the distance the shock has travelled up the wedge, then the width of the patch normal to the Mach stem scales like M^2-1 and the height scales like $\sqrt{M^2-1}$ of this length.

All the above studies, with their different interpretations and sometimes different conclusions, make it desirable to devise a very high-resolution experiment in order to resolve at least some of the issues. Over the last few years, as summarized above, the indications are clearly that there is a very small supersonic patch immediately behind the three-shock confluence, and the possibility of the existence of a multitude of such patches.

2. Apparatus

An estimate of the physical size of the patch made by Hunter & Brio (2000) using their asymptotic equations, is as follows: For a shock Mach number of 1.04, a wedge angle of 11.5° and if the shock has propagated 1 m along the wedge, the Mach stem will be 0.1 m high. The patch will then have a height of 1 mm normal to the wedge and have a width of 0.1 mm. In most conventional shock tubes a wedge length of this size is simply not achievable and the patch size will be smaller still, perhaps one-fifth of this size, corresponding to a stem length of 20 mm and a 0.2×0.02 mm patch.

Some twenty years ago the first author built a simple (constant-cross-section) shock tube 1105 mm high and 100 mm wide for the study of weak shock reflection.

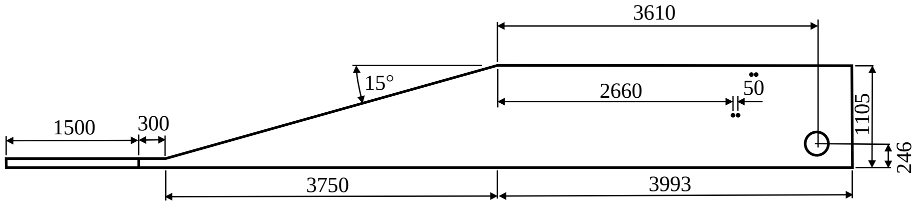

FIGURE 5. Geometry of the shock tube. Dimensions in millimetres.

Considerable difficulty was experienced in generating a plane wave due to the non-uniform rupture of the diaphragm notwithstanding many methods of initiation attempted, and the project was abandoned. With the recent important developments in the study of weak shock reflection summarized above, together with the focus on the immediate region around the three-shock confluence it was assumed that a plane incident wave was not an absolute requirement and that a cylindrical wave of large radius of curvature would serve equally well for the study of the reflection geometry. The existing tube was converted and a tapered section added as indicated in figure 5.

A simple 1.5 m long driver, of 150 mm diameter cross-section, feeding into a 100 × 100 mm, 300 mm long section is used to generate a shock which on diffraction into the tapered section generates a cylindrical wave, which reflects off a 15° wedge when its radius is about 4 m and is photographed through a window when the radius is about 7.5 m and thus is close to being plane. At that stage the Mach stem length is about 800 mm long, and the situation is nearly equivalent to testing in a conventional shock tube over 2 m high since the upper wall of the test section represents the surface of the wedge. However, the flow is not pseudo-stationary, nor is the triple-point trajectory straight but at the triple point the wave should have a similar local structure to a plane incident shock reflection. The total tube length is some 9.5 m with the window positioned towards the bottom right-hand corner. The Mach number of the incident wave is measured by transducers at mid-height of the tube, 2660 mm from the corner, shown by small solid circles in figure 5, and the incident wave will decay a little between this measurement point and the window section. The Mach numbers reported are those measured at the transducer station, since the exact decay rate of the shock has yet to be established. The decay is expected to be small, thus a measured Mach 1.05 shock would decay to about Mach 1.04 at the window section. Two further transducers are positioned higher up, near the wedge surface as shown. They measure Mach numbers about 2 % higher for the base of the Mach stem.

A 1 μs xenon flashlamp in a conventional Z-configuration schlieren system was used to photograph the waves. The knife edge was positioned to be roughly parallel to the reflected wave and for some tests was rotated by 180°. The images were recorded on 35 mm film and the negatives were scanned at 9600 d.p.i. In some experiments two cotton threads 80 mm apart were positioned across the window.

3. Results and discussion

Figure 6 shows two typical images, rotated by 90° anticlockwise relative to the conventional presentation of a Mach reflection. Thus the incident shock is at the top left and is propagating upwards. These results are for the two knife-edge orientations used and the image width corresponds to some 17 cm of the test section. Both images clearly show the existence of an expansion wave immediately behind the reflected

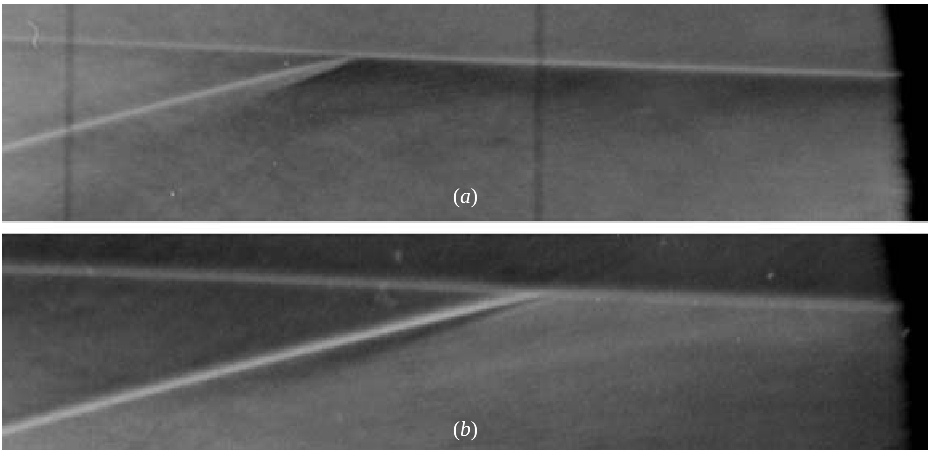

FIGURE 6. Schlieren images with opposite knife-edge positions. (a) $M = 1.084$, stem length = 798 mm, (b) $M = 1.069$, stem length = 762 mm.

wave as proposed by Guderley (1947) and shown in the recent Euler simulations of both Vasil'ev & Kraiko (1999) (see figure 2) and Zakharian *et al.* (2000). All forty images taken to date over the Mach number range tested ($M = 1.05$ to 1.1) clearly show this feature. It is thus established that the four-wave geometry with an expansion wave behind the reflected shock is a real phenomenon in weak shock wave reflection for the parameter space covered by these tests.

In addition both images show a distinct sharp contrasting line immediately after the expansion wave, indicating the existence of a terminating shock of about 15 mm length, i.e. less than 2 % of the length of the Mach stem, and which will be referred to as the first shocklet. Thus the suggestion that a shock may terminate the expansion as occurs on a transonic airfoil appears to be borne out. A fairly strong density gradient is evident behind this wave, but no distinguishing features are immediately apparent in it. In order to explore the region behind this wave in more detail all images were studied for evidence of additional features and those showing some signs were enlarged and the image contrast in the region of interest adjusted to highlight these effects. This was mainly done using the contrast enhancement adjustment in Corel Photo-Paint©.

Figure 7 shows the result for two such tests. The processed images are at double the magnification of the associated schlieren images. In the enhanced image of the first set (figure 7*a*) both the expansion wave and first shocklet are clearly evident. What is of particular interest, however, is evidence of a second shocklet behind the first in a position and orientation as predicted in the simulations of Tesdall & Hunter (2002). A similar result is noted in the second image (figure 7*b*) which shows a well-defined first shocklet. The enhanced image in this case is particularly notable for the clear evidence of two expansion waves followed by a second white area, again suggesting the existence of the second shocklet. This area is made more evident in the third image of this set where a contour line of a chosen grey value is drawn directly from the schlieren image. Some additional image processing, such as image sharpening, shows some evidence of even more structure but this is not included since such processing may introduce artifacts in the image, whereas the contrast enhancement and contouring is based only on the grey scale of each individual pixel as obtained from the original photographs.

(a)

(b)

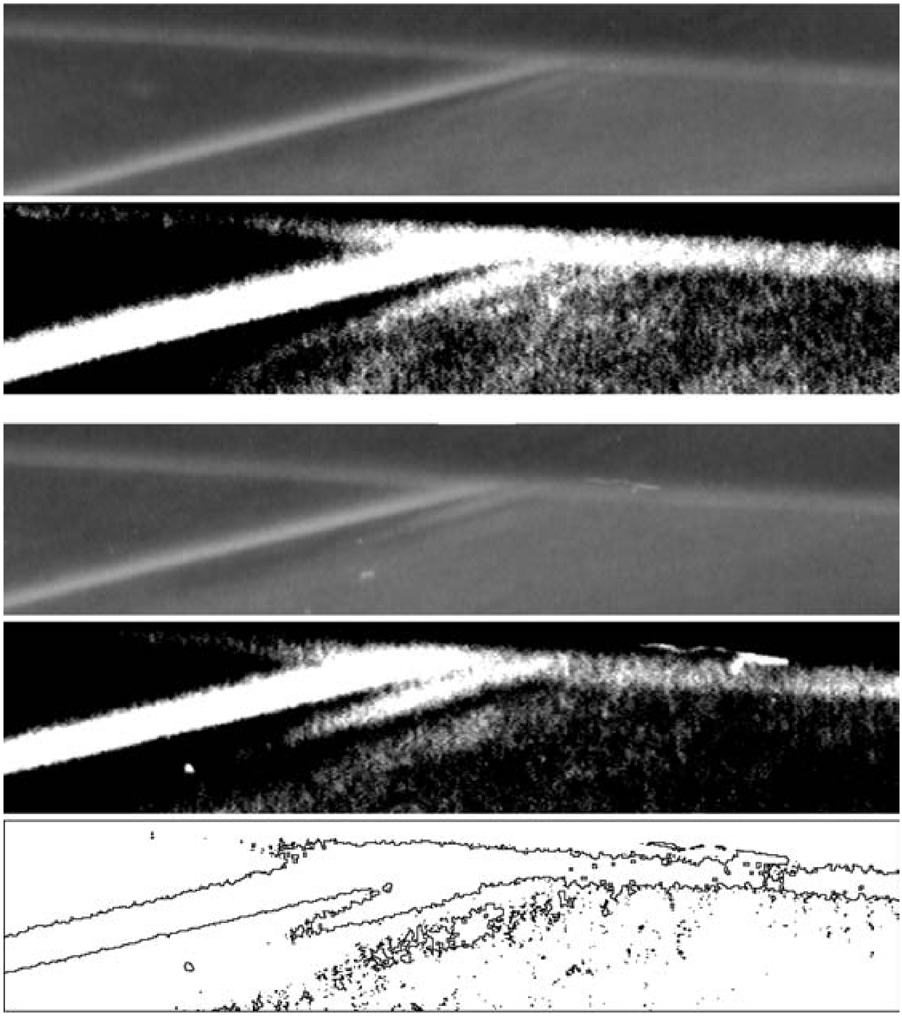

FIGURE 7. Two examples identifying the complex flow structure behind the reflected wave through image processing. (a) $M = 1.074$, stem length = 772 mm, (b) $M = 1.073$, stem length = 766 mm.

These effects are somewhat variable between images and may be sensitive to knife-edge cut-off and other factors. Nevertheless the evidence of a complex multi-wave pattern behind the reflected wave is clear. It would be desirable to implement improved diagnostics and to explore a wider range of Mach numbers, or even modify the wall angle, although the latter is a major task, to explore futher the nature and extent of these structures. It would also be advantageous to have numerical resources, and results, to determine the best range of experimental conditions to improve the resolution of the experiments, both in terms of the size of the supersonic patch, and the strength of the waves.

All the theoretical and numerical simulations done to date have assumed an inviscid flow. The effects of viscosity on these flows should be an area for further study. Zakharian *et al.* (2000) have estimated that for a Mach stem height of 1 m (similar to this investigation) the patch would be an order of magnitude larger than the reflected shock thickness. It is not known to what extent the high multiple density gradients existing in the complex flow region predicted in these inviscid models would be smoothed out due to viscous effects. For this reason some researchers have expressed doubts that they would ever be identified experimentally. Ultra-high-resolution simulations using viscous codes would thus be particularly needed.

Since the current experiments only cover a very small part of the parameter space identified in the literature as falling within the von Neumann reflection region it is not known how general such complex flows are. Because it may become necessary to distinguish between different flows within this space it may be appropriate to refer to flows where a supersonic patch exists as Guderley reflection, in recognition of his prediction of its existence over fifty years ago. The feature may also be quite common since it has been shown numerically to exist for plane incident shocks, and experimentally in this work for cylindrical shocks. It is thus likely that it will exist for spherical blast waves as well. The range for which this complex geometry occurs, and its transition boundaries, should be the subject of further study.

4. Conclusions

High-resolution experiments on the reflection of weak shock waves have shown the existence of a very small complex flow structure, similar to that predicted by Tesdall & Hunter (2002), immediately behind the three-shock confluence. The presence of an expansion wave behind the reflected shock, as originally suggested by Guderley (1947), and now also shown experimentally to exist, resolves the von Neumann Paradox.

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